University of Liège
Faculty of Applied Sciences
Urban & Environmental Engineering

University of Newcastle
Faculty of Engineering and Built Environment
Priority Research Centre for Geotechnical Science & Engineering

Hydro-mechanical modelling of multiphase flow in naturally fractured coalbeds applied to CBM recovery or CO_2 storage

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François Bertrand

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Rue d'Egmont, 5
1000 Bruxelles

And the "Wallonie-Bruxelles International" Agency

Wallonie-Bruxelles International

Place Sainctelette, 2
1080 Bruxelles

Jury:

Prof. Frédéric COLLIN	(Supervisor)	University of Liège (Belgium)
Prof. Olivier BUZZI	(Co-supervisor)	University of Newcastle (Australia)
Prof. Robert CHARLIER		University of Liège (Belgium)
Prof. Cedric GOMMES		University of Liège (Belgium)
Prof. Anne-Catherine DIEUDONNÉ		Delft University of Technology (The Netherlands)
Prof. Pierre BÉSUELLE		University of Grenoble (France)
Prof. Thierry MASSART		Free University of Brussels (Belgium)
Prof. Jean-Michel PEREIRA		Ecole des Ponts - ParisTech (France)

STATEMENT OF ORIGINALITY

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F. Bertrand

ACKNOWLEDGMENT OF AUTHORSHIP

I hereby certify that the work embodied in this thesis contains published papers of which I am a joint author. I have included as part of the thesis a written declaration endorsed in writing by my supervisor, attesting to my contribution to the joint publications.

By signing below I confirm that François Bertrand contributed to write the papers entitled *A fully coupled hydro-mechanical model for the modeling of coalbed methane recovery* (published), *Cleat-scale modelling of the coal permeability evolution due to sorption-induced strain* (under review for publication on March 2, 2020) and *Application of the FE2 method to the hydro-mechanical modelling of multiphase flow in fractured coalbed* (in preparation on March 2, 2020), for which he developed and implemented some constitutive models.

F. Collin

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Abstract

This thesis is dedicated to the modelling of multiphase flows in naturally fractured rocks and, in particular, to the recovery of methane, or reversely to the storage of carbon dioxide, in coalbeds. In this context, some hydro-mechanical couplings can likely affect the permeability of the reservoir. On the one hand, the increase in effective stress after the reservoir depletion tends to decrease the permeability. On the other hand, the matrix shrinkage following gas desorption tends to increase the permeability. These phenomena are highlighted with some experimental tests carried out in laboratory. Some numerical models are developed in this thesis to properly take into account the permeability evolution during the gas production/storage. As coal is rarely dry *in situ*, constitutive models are developed for unsaturated conditions. These models are implemented in the finite element code Lagamine.

The first model is developed at the macroscale, as generally followed in the literature for reservoir modelling. Then, fractures and matrix blocks are directly modelled with a microscale model. Particular attention is paid to the applicability of unsaturated formalism to a single fracture (modelled with an interface finite element). The numerical permeability model at the fracture scale is also compared to the analytical solution of a simple geometry. Finally, in order to model a reservoir, the modelling of the representative elementary volume is integrated in a multiscale approach with the finite element square method.

The first part of the thesis presents the context of the research. After a literature review of some remarkable experimental results, an experimental study on a Australian coal is then presented in the second part. The macroscale (reservoir scale), the microscale (laboratory scale) and the multiscale (from the laboratory to the reservoir) models are then presented in distinct parts. Finally, the last part contains the general conclusions of the thesis.

Résumé

Cette thèse est consacrée à la modélisation des écoulements multiphasiques au sein de réservoirs naturellement fracturés, plus particulièrement à la production du méthane des couches de charbon ou au stockage de dioxide de carbone dans ces veines de charbon. Dans ce contexte, des couplages hydro-mécaniques peuvent affecter la perméabilité du réservoir. D'une part, l'augmentation des contraintes effectives après une baisse de pression du réservoir tend à diminuer la perméabilité. D'autre part, le retrait de la matrice de charbon suite à la désorption du gaz tend à augmenter la perméabilité. Ces phénomènes sont mis en évidence par des essais hydro-mécaniques réalisés en laboratoire. Des modèles numériques sont dévéloppés afin de tenir compte de l'évolution de la perméabilité au cours de la production ou du stockage de gaz. A noter que le charbon est rarement sec *in situ*, les modèles constitutifs sont donc écrits en non-saturé. Ces modèles sont ensuite implémentés dans le code élément fini Lagamine.

Le premier modèle est développé à l'échelle macroscopique, comme ce qui se fait régulièrement dans la littérature pour les modélisations de réservoirs. Ensuite, un modèle microéchelle est développé pour décrire directement le comportement des fractures et des blocs matrice. Une attention particulière est portée à l'applicabilité du formalisme non-saturé à l'échelle d'une fracture unique (modélisée par un élément fini interface). Le modèle numérique de perméabilité à l'échelle de la fracture est aussi comparé à la solution analytique d'une géométrie simple. Finalement, ce modèle à l'échelle élémentaire est intégré dans une approche multi-échelle grâce à la méthode des éléments finis au carré en vue d'une modélisation à l'échelle d'un réservoir.

La première partie de la thèse présente le contexte des recherches. Ensuite, après une revue bibliographique de quelques résultats expérimentaux remarquables, la deuxième partie présente une étude expérimentale menée sur un charbon australien. Les modèles macro-échelle (échelle du réservoir), micro-échelle (échelle du laboratoire) et multi-échelle (du laboratoire au réservoir) sont ensuite présentés dans des parties distinctes. Enfin, la dernière partie contient les conclusions générales de la thèse.

Preface

The work presented in this thesis has been published or is under consideration for publication in different scientific journals. You will find below a list of these publications or expected ones. In order to improve the readability, these papers have been extended, merged and linked to constitute the thesis. Some introduction and conclusions parts are also added.

Macroscale [Bertrand et al., 2017]:

Bertrand, F., Cerfontaine, B., and Collin, F. A fully coupled hydro-mechanical model for the modeling of coalbed methane recovery. *Journal of Natural Gas Science and Engineering*. (2017).

Microscale [Bertrand et al., 2019]

Bertrand, F., Buzzi, O., and Collin, F. Cleat-scale modelling of the coal permeability evolution due to sorption-induced strain. *Journal of Coal Geology*. (2019).

Multiscale [Bertrand et al., 2020] :

Bertrand, F., Buzzi, O., Bésuelle, P., and Collin, F. Hydro-mechanical modelling of multiphase flow in naturally fractured coalbed using a multi-scale approach. *Journal of Natural Gas Science and Engineering*. (2020).

Moreover, the results of the research were also presented in the form of posters or presentations at different national and international conferences or seminars. In chronological order:

Macroscale simple-porosity model:
Efficiency of shaft sealing for CO_2 sequestration in coal mines. Workshop EAGE "Geomechanics and Energy". Celle (Germany). 12 to 15 October 2015.

Macroscale dual-porosity model, hydraulic part:
Geomechanical aspects of coalbed methane (CBM) production : Flow model formulation. RUGC 2016. Liège (Belgium). 24 to 27 May 2016.

Macroscale dual-porosity model:
Hydro-mechanical modelling of coalbed methane flows: A hypothetical reservoir example. Contact FNRS Day on "Geomechanics and Couplings". Gembloux (Belgium). 9 February 2017.

Macroscale dual-porosity model:

Simulation of coalbed methane flows, hydro-mechanical modelling in a particular fractured reservoir. 79th EAGE conference. Paris (France). 12 to 15 June 2017.

Macroscale dual-porosity model:

Hydro-mechanical modelling of a coalbed methane production well via a dual-porosity approach. GeoProc 2017. Paris (France). 5 to 7 July 2017.

Microscale direct model:

Modelling of the permeability alteration of coal due to sorption. Lagashop 2018. Delft (The Netherlands). 31 January to 2 February 2018.

Macroscale and microscale models comparison:

Modelling of the permeability evolution of coal due to sorption: Review of different scales analysis. PhD UoN Seminar. Newcastle (Australia). 23 July 2018.

Experimental results and microscale model:

Laboratory-scale study on the swelling behaviour of coal due to CO2 injection. 5th CO2 Geological Storage Workshop. Utrecht (The Netherlands). 21 to 23 November 2018.

Multiscale model:

Hydro-mechanical modelling of multiphase flow in coalbed by computational homogenization. 16th International Conference of IACMAG. Turin (Italy). 1 to 4 July 2020.

Contents

List of Symbols

Acronyms

AMM	Abandoned mine methane	
ASTM	American Society for Testing and Materials	
CBM	Coalbed methane	
CCS	Carbon dioxide Capture and Storage	
CMM	Coal mine methane	
CSG	Coal seam gas	
UCG	Underground coal gasification	

Greek Symbols

$1\ _w$	Water compressibility	$M\ ^{1}LT^{2}$
	Species	
	Linear sorption-induced strain coefficient	
	Volumetric sorption-induced strain coefficient	$ML\ ^{3}$
	Bishop's stress parameter	
	Discrete variation of a quantity	
	Infinitesimal variation of a quantity	
$_{ij}$	Kronecker symbol	
t	Time step	T
t	Sub time step	T
	Second coordinate for the parent finite element	
	External surface of the domain	$ML\ ^{1}T\ ^{2}$
	Swelling pore strain parameter	
$_0$	External reference surface of the REV	L^2
$_c$	Contact boundary between matrix blocks	L^2
$_{\overline{q}}$	Boundaries with imposed flux $\overline{q}$	L^2
$_{\tilde{q}}$	Boundaries with imposed flux $\tilde{q}$	L^2
	Geometric transmissivity function along the channel	L^3

$_{n}$	Knudsen number	
$_{ob}$	Penalty coefficient for the outer boundary flow	$L\,^{1}T\,^{1}$
	Tortuosity	
	Cleat size distribution index	
$_{m}$	First Lamé parameter of the matrix	$ML\,^{1}T\,^{2}$
μ	Dynamic viscosity	$ML\,^{1}T\,^{1}$
μ_f	Friction coefficient	
μ_g	Gas viscosity	$ML\,^{1}T\,^{1}$
μ_r	Viscosity ratio	$ML\,^{1}T\,^{1}$
μ_w	Water viscosity	$ML\,^{1}T\,^{1}$
	Poisson's ratio	
$_{m}$	Poisson's ratio of the matrix	
$_{ij}$	Poisson's ratios of the equivalent medium	
	Volume of the control space	L^3
$_{0}$	Reference volume of the REV	L^3
$_{e}$	Volume of a finite element e	L^3
$_{f}$	Volume of the fractures	L^3
$_{g}$	Gas volume	L^3
$_{g}$	Void volume	L^3
$_{g}$	Gas mass flux in the channel	$MT\,^{1}$
$_{v}$	Void volume	L^3
$_{w}$	Water mass flux	$MT\,^{1}$
	Porosity	
$_{l}$	Geometric transmissivity of the channel	L^2
$_{0}$	Initial porosity (t 0)	
$_{f}$	Porosity from fractures	
	Phase	
	Shape factor	$L\,^{2}$
	Density	$ML\,^{3}$
$_{c}$	Coal density	$ML\,^{3}$
$_{g}$	Gas density	$ML\,^{3}$
$_{s}$	Solid density	$ML\,^{3}$
$_{v}$	Water vapour density	$ML\,^{3}$
$^{0}_{v}$	Density of saturated water vapour	$ML\,^{3}$
$_{w}$	Water density	$ML\,^{3}$
$_{gf}$	Gas density in the cleats	$ML\,^{3}$

$\rho_{g,f}$	Gas density in the fracture	$[ML^{-3}]$
$\rho_{g,f}^{d}$	Density of dissolved gas in water	$[ML^{-3}]$
$\rho_{g,std}$	Gas density at standard conditions	$[ML^{-3}]$
ρ_{g0}	Reference gas density	$[ML^{-3}]$
ρ_{g}^{Ad}	Density of gas adsorbed on the matrix	$[ML^{-3}]$
ρ_{w0}	Reference water density	$[ML^{-3}]$
σ_{ij0}	Initial stresses	$[ML^{-1}T^{-2}]$
σ_{ij}	Cauchy stress tensor	$[ML^{-1}T^{-2}]$
σ'_{ij}	Effective stress tensor	$[ML^{-1}T^{-2}]$
τ	Shear stress	$[ML^{-1}T^{-2}]$
$\tau_{c,w}$	Channel tortuosity of the water phase	$[-]$
τ_{cgw}	Channel tortuosity of the gas phase	$[-]$
Θ	Number of sites covered by adsorbed molecules	
θ	Angle	$[-]$
$\tilde{\sigma_{ij}}$	Jaumann stress rate	$[ML^{-1}T^{-3}]$
ε	Small perturbation	
ε_{b}	Bulk strain	$[-]$
ε_{p}	Pore strain	$[-]$
ε_{b_s}	Bulk sorption-induced strain	$[-]$
ε_{ij}	Strain tensor	$[-]$
ε_{p_s}	Pore sorption-induced strain	$[-]$
Ξ	Volume fraction	$[-]$
ξ	First coordinate for the parent finite element	
Ξ_{g}	Gas volume fraction	$[-]$
Ξ_{l}	Liquid volume fraction	$[-]$
Ξ_{s}	Solid volume fraction	$[-]$
ζ	Tortuosity parameter	$[-]$
ε_{vs}	Volumetric sorption-induced strain	$[-]$
$\varepsilon_{(ii)_s}$	Linear sorption-induced strain in the direction i	$[-]$

Roman Symbols

$[A]$	Matrix of partial derivatives of coordinates for a parent continuum element
$[A^{I}]$	Matrix of partial derivatives of coordinates for an interface element
$[B]$	Matrix of partial derivatives of shape functions for a parent continuum element

$[B^I]$	Matrix of partial derivatives of shape functions for an interface element	
$[C_{mm}]$	Mechanical constitutive matrix	$[ML^{-1}T^{-2}]$
$[K_{mm}]$	Mechanical stiffness matrix	$[ML^{-1}T^{-2}]$
$\Delta\mathcal{H}$	Differential enthalpy of adsorption	$[ML^2T^{-2}N^{-1}]$
$\Delta\mathcal{S}^0$	Standard molar integral entropy at saturation	$[ML^2T^{-2}N^{-1}\theta^{-1}]$
$\mathcal{A}$	Adsorbed gas content parameter	$[-]$
$\mathcal{C}_g$	Integration constant	$[T^{-1}]$
$\mathcal{C}_w$	Integration constant	$[T^{-1}]$
$\mathcal{K}$	Some internal variables	
$\mathcal{M}_g$	Gas molecules	
$\mathcal{N}$	Interpolation function	
$O-$	Vacant surface sites	
$\mathcal{P}$	Point with coordinates x_i	
$\mathcal{T}$	Sorption time	$[T]$
$\mathcal{Z}$	Integration constant	$[LT^{-1}]$
$\mathcal{Z}_g$	Integration constant	$[LT^{-1}]$
$\mathfrak{N}$	Power-law function describing a fractal distribution	
$\bar{l}$	Gas mean free path	$[L]$
$\bar{q}$	Boundary flow	$[LT^{-1}]$
$\bar{q}_g$	Gas boundary flow	$[LT^{-1}]$
$\bar{q}_w$	Water boundary flow	$[LT^{-1}]$
$\bar{t}_i$	External traction force	$[ML^{-1}T^{-2}]$
$\{f\}$	Global nodal force components	
$\{u\}$	Global nodal displacement components	
$\{U^{Node}\}$	Column vectors of nodal displacements	$[L]$
$\{V^{Node}\}$	Column vectors of nodal velocities	$[LT^{-1}]$
$\{X^{Node}\}$	Column vectors of nodal positions	$[L]$
A	Area or Boundary surface area	$[L^2]$
a	Constant of proportionality	$[-]$
a'	Constant of proportionality	$[-]$
a''	Constant of proportionality	$[-]$
b	Biot's coefficient	$[-]$
b_g	Klinkenberg number	$[-]$
b_{ij}	Biot's coefficient tensor	$[-]$
C	Concentration	$[NL^{-3}]$

c	Kundt and Warburg's constant	$[-]$
C_g	Gas concentration	$[NL^{-3}]$
C_{ijkl}	Constitutive mechanical (stiffness) tensor	$[ML^{-1}T^{-2}]$
D_f	Fractal dimension	$[-]$
d_g	Collision diameter of a gas molecule	$[L]$
d_p	Pore diameter	$[L]$
D^{α}_{β}	Diffusion coefficient of the species α through β	$[L^2T^{-1}]$
$D^{\alpha *}_{\beta}$	Effective diffusion coefficient of the species α through β	$[L^2T^{-1}]$
D_{ijkl}	Compliance tensor	$[M^{-1}LT^2]$
E	Mass exchange between matrix blocks and fractures	$[MT^{-1}]$
E_i	Young's moduli of the equivalent medium	$[ML^{-1}T^{-2}]$
E_m	Young's modulus of the matrix	$[ML^{-1}T^{-2}]$
F_i	Force vector	$[ML^{-2}T^{-2}]$
f_i	Flux	$[ML^{-2}T^{-1}]$
F_E	Energetically equivalent external nodal forces	
f_{g_i}	Internal total flux of gas	$[ML^{-2}T^{-1}]$
f_{g_L}	Longitudinal gas mass flux	$[ML^{-2}T^{-1}]$
$f^k_{g_T}$	Transverse gas mass flux	$[ML^{-2}T^{-1}]$
F_{ij}	Deformation gradient tensor	$[-]$
F_I	Energetically equivalent internal nodal forces	
F_{OB}	Out of balance forces	
f_{w_i}	Internal total flux of water	$[ML^{-2}T^{-1}]$
f_{w_L}	Longitudinal water mass flux	$[ML^{-2}T^{-1}]$
G_m	Shear modulus of the matrix blocks	$[ML^{-1}T^{-2}]$
G_{ij}	Shear moduli of the equivalent medium	$[ML^{-1}T^{-2}]$
H	Height	$[L]$
h	Fracture aperture	$[L]$
h^{min}	Minimum fracture aperture	$[L]$
h_b	Hydraulic fracture aperture	$[L]$
H_g	Henry's coefficient	$[-]$
h_g	Height of the gas stratum in the fracture	$[L]$
h_w	Height of the water stratum in the fracture	$[L]$
$J^g_{g_i}$	Diffusive mass flux of gas in the gas phase	$[ML^{-2}T^{-1}]$
$J^w_{g_i}$	Diffusive mass flux of water vapour	$[ML^{-2}T^{-1}]$
J_{ij}	Jacobian matrix	$[-]$

$J^{g}_{l_i}$	Diffusive mass flux of dissolved gas in the liquid phase	$ML\ ^{2}T\ ^{1}$
$J^{g}_{m_i}$	Diffusive mass flux of gas in the matrix	$ML\ ^{2}T\ ^{1}$
K	Global stiffness matrix	
k	Permeability	L^{2}
k_0	Initial permeability (t 0)	L^{2}
k_B	Boltzmann constant	$ML^{2}T\ ^{2}\ \ ^{1}$
K_c	Equilibrium constant of a reaction	
k_e	Effective intrinsic permeability	L^{2}
K_m	Bulk modulus of the matrix blocks	$ML\ ^{1}T\ ^{2}$
K_n	Normal stiffness of the fracture	$ML\ ^{2}T\ ^{2}$
K_n^0	Normal stiffness of the fracture for zero-displacement	$ML\ ^{2}T\ ^{2}$
K_p	Cleat stiffness	$ML\ ^{1}T\ ^{2}$
K_s	Shear stiffness of the fracture	$ML\ ^{2}T\ ^{2}$
k_{cleat}	Cleat permeability	L^{2}
k_{rg}	Relative permeability to gas	
k_{rw}	Relative permeability to water	
L	Fracture length	L
l	Width of the contact zone	L
L_c	Macroscopic characteristic length	L
l_c	Microscopic characteristic length	L
l_u	Length of a capillary tube	L
L_{ij}	Velocity gradient field	$T\ ^{1}$
l_{REV}	Size of the REV	L
M	Mass	M
m	Material	L
M_g	Gas mass content	M
M_m	P-wave modulus of the matrix	$ML\ ^{1}T\ ^{2}$
M_w	Water mass content	M
$M^{d}_{g\ f}$	Gas mass dissolved in the water in the fracture	M
$M^{g}_{g\ f}$	Gas mass in the gas phase in the fracture	M
$M^{Ad}_{g\ m}$	Gas mass adsorbed in the matrix	M
M_{m_g}	Gas molecular mass	$MN\ ^{1}$
M_{m_w}	Water molecular mass	$MN\ ^{1}$
N	Number of sets of fractures	
N_i	Unit vector normal to the surface of the REV	

n_i	Unit vector normal to the boundary	
n_{rg}	Exponent parameter for the stauration degree formulation	
n_{rw}	Exponent parameter for the stauration degree formulation	
p	Pressure	$ML^{1}T^{2}$
p_0	Initial pressure (t 0)	$ML^{1}T^{2}$
p_a	Atmospheric pressure	$ML^{1}T^{2}$
p_c	Capillary pressure	$ML^{1}T^{2}$
p_e	Entry capillary pressure	$ML^{1}T^{2}$
p_f	Fracture pressure	$ML^{1}T^{2}$
p_g	Gas pressure	$ML^{1}T^{2}$
p_g	Virtual gas pressure	$ML^{1}T^{2}$
p_g^f	Fluctuation of gas pressure	$ML^{1}T^{2}$
P_L	Langmuir pressure	$ML^{1}T^{2}$
p_m	Matrix pressure	$ML^{1}T^{2}$
p_w	Water pressure	$ML^{1}T^{2}$
p_w	Virtual water pressure	$ML^{1}T^{2}$
p_w^f	Fluctuation of water pressure	$ML^{1}T^{2}$
$p_{g\,f}$	Gas pressure in the fractures	$ML^{1}T^{2}$
$p_{g\,m}$	Gas pressure in the matrix	$ML^{1}T^{2}$
$p_{g\,m}^0$	Initial gas pressure in the matrix	$ML^{1}T^{2}$
$p_{g\,m}^{lim}$	Limit gas pressure	$ML^{1}T^{2}$
$p_{g\,m}^{max}$	Maximum gas pressure in the matrix	$ML^{1}T^{2}$
$p_g^{Ad^{lim}}$	Limit adsorbed gas pressure	$ML^{1}T^{2}$
p_g^{Ad}	Adsorbed gas pressure in the matrix	$ML^{1}T^{2}$
$p_g^{Ad^b}$	Adsorbed gas pressure in equilibrium with the fracture pressure	$ML^{1}T^{2}$
P_{ij}	First Piola-Kirchhoff stress tensor	$ML^{1}T^{2}$
p_{rb}	Rebound pressure	$ML^{1}T^{2}$
p_{res}	Reservoir pressure	$ML^{1}T^{2}$
p_{res}^{crit}	Critical reservoir pressure	$ML^{1}T^{2}$
p_{w_0}	Reference water pressure	$ML^{1}T^{2}$
Q	Source term	$ML^{3}T^{1}$
q	Flow	LT^{1}
q_f	Flow between two parallel plates	LT^{1}
Q_g	Gas source term	$ML^{3}T^{1}$

q_g	Input gas flux	$[ML^{-2}T^{-1}]$
q_i	Flow vector	$[LT^{-1}]$
q_L	Longitudinal flow	$[LT^{-1}]$
q_N	Total flow through N fractures	$[LT^{-1}]$
q_T	Transverse flow	$[LT^{-1}]$
Q_w	Water source term	$[ML^{-3}T^{-1}]$
q_w	Input water flux	$[ML^{-2}T^{-1}]$
q_{g_i}	Advective flow vector of the gas phase	$[LT^{-1}]$
q_{g_L}	Longitudinal flow of the gas phase	$[LT^{1}]$
q_{g_T}	Gas transverse flow	$[LT^{-1}]$
$q_{g_{well}}$	Mass gas production rate	$[ML^{-2}T^{-1}]$
q_{l_i}	Advective flow vector of the liquid phase	$[LT^{-1}]$
q_{l_L}	Longitudinal flow of the liquid phase	$[LT^{1}]$
$q_{w_{ob}}$	Income water mass flow on the outer boundary	$[ML^{-2}T^{-1}]$
$q_{w_{well}}$	Mass water production rate	$[ML^{-2}T^{-1}]$
R	Universal gas constant	$[ML^2N^{-1}\theta^{-1}T^{-2}]$
r	Radius	$[L]$
R_{ij}	Rotation matrix	$[-]$
s	Coordinate along the channel	$[L]$
S_g	Gas mass storage term	$[ML^{-2}]$
S_r	Saturation degree	$[-]$
S_r^*	Normalized saturation	$[-]$
S_w	Water mass storage term	$[ML^{-2}]$
$S_{r,res}$	Residual saturation	$[-]$
$S_{r_g,res}$	Gas residual saturation degree	$[-]$
S_{r_g}	Gas saturation degree	$[-]$
T	Temperature	$[\theta]$
t	Time	$[T]$
T_i	Projection of the local stress tensor in global coordinates	$[ML^{-1}T^{-2}]$
t_i	Traction vector	$[ML^{-1}T^{-2}]$
T_t	Transverse transmissivity of the fracture	$[M^{-1}L^2T^{1}]$
T_{well}	Transmissibility factor of the well	$[L^3]$
u_i	Displacement vector	$[L]$
u_i^f	Fluctuation displacement field	$[L]$
$u_i^{\wr}$	Equivalent displacement in the contact zone	$[L]$

u_n	Normal displacement	$[L]$
u_n^{max}	Maximal normal displacement allowed	$[L]$
u_{l_k}	Coordinate of the degree of freedom l at node k	
V	Volume	$[L^3]$
v_i	Velocity vector	$[LT^{-1}]$
v_i^*	Admissible virtual velocity field	$[LT^{-1}]$
V_L	Langmuir volume	$[L^3M^{-1}]$
v_g	Gas molecular velocity	$[LT^{-1}]$
V_g^{Ad}	Adsorbed volume per unit of mass	$[L^3M^{-1}]$
w	Matrix width	$[L]$
W_E^*	External virtual work	$[ML^2T^{-2}]$
W_G	Gauss weight at the integration point IP	
W_I^*	Internal virtual work	$[ML^2T^{-2}]$
w_{ij}	Spin rate tensor	T^{-1}
X_i	Coordinates in the reference configuration	$[L]$
x_i	Coordinates in the current configuration	$[L]$

Superscripts

$[.]^e$	Quantity related to a finite element e
$[.]^F$	Quantity on the follow boundary
$[.]^L$	Quantity on the lead boundary
$[.]^M$	Macroscale quantity
$[.]^m$	Microscale quantity
$[.]^T$	Transposed object
$[\dot{.}]$	Time derivative
$^{\diamond}[.]$	Quantity given in the orthotropic axes

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Dubito, ergo sum. Cogito, ergo sum.

René Descartes