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**Hydro-mechanical modelling of  
multiphase flow in naturally fractured coalbeds  
applied to CBM recovery or  $CO_2$  storage**

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Thesis submitted for the degree of  
*Philosophiae Doctor* in Applied Sciences

Presented by

**François BERTRAND**

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F. Bertrand

## ACKNOWLEDGMENT OF AUTHORSHIP

I hereby certify that the work embodied in this thesis contains published papers of which I am a joint author. I have included as part of the thesis a written declaration endorsed in writing by my supervisor, attesting to my contribution to the joint publications.

By signing below I confirm that François Bertrand contributed to write the papers entitled *A fully coupled hydro-mechanical model for the modeling of coalbed methane recovery* (published), *Cleat-scale modelling of the coal permeability evolution due to sorption-induced strain* (under review for publication on March 2, 2020) and *Application of the FE2 method to the hydro-mechanical modelling of multiphase flow in fractured coalbed* (in preparation on March 2, 2020), for which he developed and implemented some constitutive models.

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# Abstract

This thesis is dedicated to the modelling of multiphase flows in naturally fractured rocks and, in particular, to the recovery of methane, or reversely to the storage of carbon dioxide, in coalbeds. In this context, some hydro-mechanical couplings can likely affect the permeability of the reservoir. On the one hand, the increase in effective stress after the reservoir depletion tends to decrease the permeability. On the other hand, the matrix shrinkage following gas desorption tends to increase the permeability. These phenomena are highlighted with some experimental tests carried out in laboratory. Some numerical models are developed in this thesis to properly take into account the permeability evolution during the gas production/storage. As coal is rarely dry *in situ*, constitutive models are developed for unsaturated conditions. These models are implemented in the finite element code Lagamine.

The first model is developed at the macroscale, as generally followed in the literature for reservoir modelling. Then, fractures and matrix blocks are directly modelled with a microscale model. Particular attention is paid to the applicability of unsaturated formalism to a single fracture (modelled with an interface finite element). The numerical permeability model at the fracture scale is also compared to the analytical solution of a simple geometry. Finally, in order to model a reservoir, the modelling of the representative elementary volume is integrated in a multiscale approach with the finite element square method.

The first part of the thesis presents the context of the research. After a literature review of some remarkable experimental results, an experimental study on a Australian coal is then presented in the second part. The macroscale (reservoir scale), the microscale (laboratory scale) and the multiscale (from the laboratory to the reservoir) models are then presented in distinct parts. Finally, the last part contains the general conclusions of the thesis.



# Résumé

Cette thèse est consacrée à la modélisation des écoulements multiphasiques au sein de réservoirs naturellement fracturés, plus particulièrement à la production du méthane des couches de charbon ou au stockage de dioxyde de carbone dans ces veines de charbon. Dans ce contexte, des couplages hydro-mécaniques peuvent affecter la perméabilité du réservoir. D'une part, l'augmentation des contraintes effectives après une baisse de pression du réservoir tend à diminuer la perméabilité. D'autre part, le retrait de la matrice de charbon suite à la désorption du gaz tend à augmenter la perméabilité. Ces phénomènes sont mis en évidence par des essais hydro-mécaniques réalisés en laboratoire. Des modèles numériques sont développés afin de tenir compte de l'évolution de la perméabilité au cours de la production ou du stockage de gaz. A noter que le charbon est rarement sec *in situ*, les modèles constitutifs sont donc écrits en non-saturé. Ces modèles sont ensuite implémentés dans le code élément fini Lagamine.

Le premier modèle est développé à l'échelle macroscopique, comme ce qui se fait régulièrement dans la littérature pour les modélisations de réservoirs. Ensuite, un modèle microéchelle est développé pour décrire directement le comportement des fractures et des blocs matrice. Une attention particulière est portée à l'applicabilité du formalisme non-saturé à l'échelle d'une fracture unique (modélisée par un élément fini interface). Le modèle numérique de perméabilité à l'échelle de la fracture est aussi comparé à la solution analytique d'une géométrie simple. Finalement, ce modèle à l'échelle élémentaire est intégré dans une approche multi-échelle grâce à la méthode des éléments finis au carré en vue d'une modélisation à l'échelle d'un réservoir.

La première partie de la thèse présente le contexte des recherches. Ensuite, après une revue bibliographique de quelques résultats expérimentaux remarquables, la deuxième partie présente une étude expérimentale menée sur un charbon australien. Les modèles macro-échelle (échelle du réservoir), micro-échelle (échelle du laboratoire) et multi-échelle (du laboratoire au réservoir) sont ensuite présentés dans des parties distinctes. Enfin, la dernière partie contient les conclusions générales de la thèse.



# Preface

The work presented in this thesis has been published or is under consideration for publication in different scientific journals. You will find below a list of these publications or expected ones. In order to improve the readability, these papers have been extended, merged and linked to constitute the thesis. Some introduction and conclusions parts are also added.

Macroscale [Bertrand et al., 2017]:

Bertrand, F., Cerfontaine, B., and Collin, F. A fully coupled hydro-mechanical model for the modeling of coalbed methane recovery. *Journal of Natural Gas Science and Engineering*. (2017).

Microscale [Bertrand et al., 2019]

Bertrand, F., Buzzi, O., and Collin, F. Cleat-scale modelling of the coal permeability evolution due to sorption-induced strain. *Journal of Coal Geology*. (2019).

Multiscale [Bertrand et al., 2020] :

Bertrand, F., Buzzi, O., Bésuelle, P., and Collin, F. Hydro-mechanical modelling of multiphase flow in naturally fractured coalbed using a multi-scale approach. *Journal of Natural Gas Science and Engineering*. (2020).

Moreover, the results of the research were also presented in the form of posters or presentations at different national and international conferences or seminars. In chronological order:

Macroscale simple-porosity model:

Efficiency of shaft sealing for  $CO_2$  sequestration in coal mines. Workshop EAGE "Geomechanics and Energy". Celle (Germany). 12 to 15 October 2015.

Macroscale dual-porosity model, hydraulic part:

Geomechanical aspects of coalbed methane (CBM) production : Flow model formulation. RUGC 2016. Liège (Belgium). 24 to 27 May 2016.

Macroscale dual-porosity model:

Hydro-mechanical modelling of coalbed methane flows: A hypothetical reservoir example. Contact FNRS Day on "Geomechanics and Couplings". Gembloux (Belgium). 9 February 2017.

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Macroscale dual-porosity model:

Simulation of coalbed methane flows, hydro-mechanical modelling in a particular fractured reservoir. 79th EAGE conference. Paris (France). 12 to 15 June 2017.

Macroscale dual-porosity model:

Hydro-mechanical modelling of a coalbed methane production well via a dual-porosity approach. GeoProc 2017. Paris (France). 5 to 7 July 2017.

Microscale direct model:

Modelling of the permeability alteration of coal due to sorption. Lagashop 2018. Delft (The Netherlands). 31 January to 2 February 2018.

Macroscale and microscale models comparison:

Modelling of the permeability evolution of coal due to sorption: Review of different scales analysis. PhD UoN Seminar. Newcastle (Australia). 23 July 2018.

Experimental results and microscale model:

Laboratory-scale study on the swelling behaviour of coal due to CO<sub>2</sub> injection. 5th CO<sub>2</sub> Geological Storage Workshop. Utrecht (The Netherlands). 21 to 23 November 2018.

Multiscale model:

Hydro-mechanical modelling of multiphase flow in coalbed by computational homogenization. 16th International Conference of IACMAG. Turin (Italy). 1 to 4 July 2020.

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# List of Symbols

## *Acronyms*

AMM	Abandoned mine methane
ASTM	American Society for Testing and Materials
CBM	Coalbed methane
CCS	Carbon dioxide Capture and Storage
CMM	Coal mine methane
CSG	Coal seam gas
UCG	Underground coal gasification

## *Greek Symbols*

$\Gamma_w$	Water compressibility	$M^{-1}LT^2$
	Species	
	Linear sorption-induced strain coefficient	
	Volumetric sorption-induced strain coefficient	$ML^{-3}$
	Bishop's stress parameter	
	Discrete variation of a quantity	
	Infinitesimal variation of a quantity	
$\delta_{ij}$	Kronecker symbol	
$t$	Time step	$T$
$t_s$	Sub time step	$T$
	Second coordinate for the parent finite element	
	External surface of the domain	$ML^{-1}T^{-2}$
	Swelling pore strain parameter	
$\Gamma_0$	External reference surface of the REV	$L^2$
$\Gamma_c$	Contact boundary between matrix blocks	$L^2$
$\bar{q}$	Boundaries with imposed flux $\bar{q}$	$L^2$
$\tilde{q}$	Boundaries with imposed flux $\tilde{q}$	$L^2$
	Geometric transmissivity function along the channel	$L^3$

## LIST OF SYMBOLS

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$n$	Knudsen number	
$ob$	Penalty coefficient for the outer boundary flow	$L^{-1}T^{-1}$
	Tortuosity	
	Cleat size distribution index	
$m$	First Lamé parameter of the matrix	$ML^{-1}T^{-2}$
$\mu$	Dynamic viscosity	$ML^{-1}T^{-1}$
$\mu_f$	Friction coefficient	
$\mu_g$	Gas viscosity	$ML^{-1}T^{-1}$
$\mu_r$	Viscosity ratio	$ML^{-1}T^{-1}$
$\mu_w$	Water viscosity	$ML^{-1}T^{-1}$
	Poisson's ratio	
$m$	Poisson's ratio of the matrix	
$ij$	Poisson's ratios of the equivalent medium	
	Volume of the control space	$L^3$
$0$	Reference volume of the REV	$L^3$
$e$	Volume of a finite element $e$	$L^3$
$f$	Volume of the fractures	$L^3$
$g$	Gas volume	$L^3$
$g$	Void volume	$L^3$
$g$	Gas mass flux in the channel	$MT^{-1}$
$v$	Void volume	$L^3$
$w$	Water mass flux	$MT^{-1}$
	Porosity	
$l$	Geometric transmissivity of the channel	$L^2$
$0$	Initial porosity ( $t = 0$ )	
$f$	Porosity from fractures	
	Phase	
	Shape factor	$L^{-2}$
	Density	$ML^{-3}$
$c$	Coal density	$ML^{-3}$
$g$	Gas density	$ML^{-3}$
$s$	Solid density	$ML^{-3}$
$v$	Water vapour density	$ML^{-3}$
$0_v$	Density of saturated water vapour	$ML^{-3}$
$w$	Water density	$ML^{-3}$
$g_f$	Gas density in the cleats	$ML^{-3}$

$\rho_{g,f}$	Gas density in the fracture	$[ML^{-3}]$
$\rho_{g,f}^d$	Density of dissolved gas in water	$[ML^{-3}]$
$\rho_{g,std}$	Gas density at standard conditions	$[ML^{-3}]$
$\rho_{g0}$	Reference gas density	$[ML^{-3}]$
$\rho_g^{Ad}$	Density of gas adsorbed on the matrix	$[ML^{-3}]$
$\rho_{w0}$	Reference water density	$[ML^{-3}]$
$\sigma_{ij0}$	Initial stresses	$[ML^{-1}T^{-2}]$
$\sigma_{ij}$	Cauchy stress tensor	$[ML^{-1}T^{-2}]$
$\sigma'_{ij}$	Effective stress tensor	$[ML^{-1}T^{-2}]$
$\tau$	Shear stress	$[ML^{-1}T^{-2}]$
$\tau_{c,w}$	Channel tortuosity of the water phase	$[ - ]$
$\tau_{cgw}$	Channel tortuosity of the gas phase	$[ - ]$
$\Theta$	Number of sites covered by adsorbed molecules	
$\theta$	Angle	$[ - ]$
$\tilde{\sigma}_{ij}$	Jaumann stress rate	$[ML^{-1}T^{-3}]$
$\varepsilon$	Small perturbation	
$\varepsilon_b$	Bulk strain	$[ - ]$
$\varepsilon_p$	Pore strain	$[ - ]$
$\varepsilon_{bs}$	Bulk sorption-induced strain	$[ - ]$
$\varepsilon_{ij}$	Strain tensor	$[ - ]$
$\varepsilon_{ps}$	Pore sorption-induced strain	$[ - ]$
$\Xi$	Volume fraction	$[ - ]$
$\xi$	First coordinate for the parent finite element	
$\Xi_g$	Gas volume fraction	$[ - ]$
$\Xi_l$	Liquid volume fraction	$[ - ]$
$\Xi_s$	Solid volume fraction	$[ - ]$
$\zeta$	Tortuosity parameter	$[ - ]$
$\varepsilon_{vs}$	Volumetric sorption-induced strain	$[ - ]$
$\varepsilon_{(ii)s}$	Linear sorption-induced strain in the direction $i$	$[ - ]$

### Roman Symbols

$[A]$	Matrix of partial derivatives of coordinates for a parent continuum element
$[A^I]$	Matrix of partial derivatives of coordinates for an interface element
$[B]$	Matrix of partial derivatives of shape functions for a parent continuum element

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$[B']$	Matrix of partial derivatives of shape functions for an interface element	
$[C_{mm}]$	Mechanical constitutive matrix	$[ML^{-1}T^{-2}]$
$[K_{mm}]$	Mechanical stiffness matrix	$[ML^{-1}T^{-2}]$
$\Delta\mathcal{H}$	Differential enthalpy of adsorption	$[ML^2T^{-2}N^{-1}]$
$\Delta S^0$	Standard molar integral entropy at saturation	$[ML^2T^{-2}N^{-1}\theta^{-1}]$
$\mathcal{A}$	Adsorbed gas content parameter	$[ - ]$
$C_g$	Integration constant	$[T^{-1}]$
$C_w$	Integration constant	$[T^{-1}]$
$\mathcal{K}$	Some internal variables	
$\mathcal{M}_g$	Gas molecules	
$\mathcal{N}$	Interpolation function	
$O-$	Vacant surface sites	
$P$	Point with coordinates $x_i$	
$T$	Sorption time	$[T]$
$Z$	Integration constant	$[LT^{-1}]$
$Z_g$	Integration constant	$[LT^{-1}]$
$\mathfrak{N}$	Power-law function describing a fractal distribution	
$\bar{l}$	Gas mean free path	$[L]$
$\bar{q}$	Boundary flow	$[LT^{-1}]$
$\bar{q}_g$	Gas boundary flow	$[LT^{-1}]$
$\bar{q}_w$	Water boundary flow	$[LT^{-1}]$
$\bar{t}_i$	External traction force	$[ML^{-1}T^{-2}]$
$\{f\}$	Global nodal force components	
$\{u\}$	Global nodal displacement components	
$\{U^{Node}\}$	Column vectors of nodal displacements	$[L]$
$\{V^{Node}\}$	Column vectors of nodal velocities	$[LT^{-1}]$
$\{X^{Node}\}$	Column vectors of nodal positions	$[L]$
$A$	Area or Boundary surface area	$[L^2]$
$a$	Constant of proportionality	$[ - ]$
$a'$	Constant of proportionality	$[ - ]$
$a''$	Constant of proportionality	$[ - ]$
$b$	Biot's coefficient	$[ - ]$
$b_g$	Klinkenberg number	$[ - ]$
$b_{ij}$	Biot's coefficient tensor	$[ - ]$
$C$	Concentration	$[NL^{-3}]$

$c$	Kundt and Warburg's constant	[−]
$C_g$	Gas concentration	[ $NL^{-3}$ ]
$C_{ijkl}$	Constitutive mechanical (stiffness) tensor	[ $ML^{-1}T^{-2}$ ]
$D_f$	Fractal dimension	[−]
$d_g$	Collision diameter of a gas molecule	[ $L$ ]
$d_p$	Pore diameter	[ $L$ ]
$D_\beta^\alpha$	Diffusion coefficient of the species $\alpha$ through $\beta$	[ $L^2T^{-1}$ ]
$D_\beta^\alpha^*$	Effective diffusion coefficient of the species $\alpha$ through $\beta$	[ $L^2T^{-1}$ ]
$D_{ijkl}$	Compliance tensor	[ $M^{-1}LT^2$ ]
$E$	Mass exchange between matrix blocks and fractures	[ $MT^{-1}$ ]
$E_i$	Young's moduli of the equivalent medium	[ $ML^{-1}T^{-2}$ ]
$E_m$	Young's modulus of the matrix	[ $ML^{-1}T^{-2}$ ]
$F_i$	Force vector	[ $ML^{-2}T^{-2}$ ]
$f_i$	Flux	[ $ML^{-2}T^{-1}$ ]
$F_E$	Energetically equivalent external nodal forces	
$f_{gi}$	Internal total flux of gas	[ $ML^{-2}T^{-1}$ ]
$f_{gL}$	Longitudinal gas mass flux	[ $ML^{-2}T^{-1}$ ]
$f_{gr}^k$	Transverse gas mass flux	[ $ML^{-2}T^{-1}$ ]
$F_{ij}$	Deformation gradient tensor	[−]
$F_I$	Energetically equivalent internal nodal forces	
$F_{OB}$	Out of balance forces	
$f_{wi}$	Internal total flux of water	[ $ML^{-2}T^{-1}$ ]
$f_{wL}$	Longitudinal water mass flux	[ $ML^{-2}T^{-1}$ ]
$G_m$	Shear modulus of the matrix blocks	[ $ML^{-1}T^{-2}$ ]
$G_{ij}$	Shear moduli of the equivalent medium	[ $ML^{-1}T^{-2}$ ]
$H$	Height	[ $L$ ]
$h$	Fracture aperture	[ $L$ ]
$h^{min}$	Minimum fracture aperture	[ $L$ ]
$h_b$	Hydraulic fracture aperture	[ $L$ ]
$H_g$	Henry's coefficient	[−]
$h_g$	Height of the gas stratum in the fracture	[ $L$ ]
$h_w$	Height of the water stratum in the fracture	[ $L$ ]
$J_{gi}^g$	Diffusive mass flux of gas in the gas phase	[ $ML^{-2}T^{-1}$ ]
$J_{gi}^w$	Diffusive mass flux of water vapour	[ $ML^{-2}T^{-1}$ ]
$J_{ij}$	Jacobian matrix	[−]

## LIST OF SYMBOLS

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$J_{l_i}^g$	Diffusive mass flux of dissolved gas in the liquid phase	$ML^{-2}T^{-1}$
$J_{m_i}^g$	Diffusive mass flux of gas in the matrix	$ML^{-2}T^{-1}$
$K$	Global stiffness matrix	
$k$	Permeability	$L^2$
$k_0$	Initial permeability ( $t = 0$ )	$L^2$
$k_B$	Boltzmann constant	$ML^2T^{-2}-1$
$K_c$	Equilibrium constant of a reaction	
$k_e$	Effective intrinsic permeability	$L^2$
$K_m$	Bulk modulus of the matrix blocks	$ML^{-1}T^{-2}$
$K_n$	Normal stiffness of the fracture	$ML^{-2}T^{-2}$
$K_n^0$	Normal stiffness of the fracture for zero-displacement	$ML^{-2}T^{-2}$
$K_p$	Cleat stiffness	$ML^{-1}T^{-2}$
$K_s$	Shear stiffness of the fracture	$ML^{-2}T^{-2}$
$k_{cleat}$	Cleat permeability	$L^2$
$k_{rg}$	Relative permeability to gas	
$k_{rw}$	Relative permeability to water	
$L$	Fracture length	$L$
$l$	Width of the contact zone	$L$
$L_c$	Macroscopic characteristic length	$L$
$l_c$	Microscopic characteristic length	$L$
$l_u$	Length of a capillary tube	$L$
$L_{ij}$	Velocity gradient field	$T^{-1}$
$l_{REV}$	Size of the REV	$L$
$M$	Mass	$M$
$m$	Material	$L$
$M_g$	Gas mass content	$M$
$M_m$	P-wave modulus of the matrix	$ML^{-1}T^{-2}$
$M_w$	Water mass content	$M$
$M_{g_f}^d$	Gas mass dissolved in the water in the fracture	$M$
$M_{g_f}^s$	Gas mass in the gas phase in the fracture	$M$
$M_{g_m}^{Ad}$	Gas mass adsorbed in the matrix	$M$
$M_{m_g}$	Gas molecular mass	$MN^{-1}$
$M_{m_w}$	Water molecular mass	$MN^{-1}$
$N$	Number of sets of fractures	
$N_i$	Unit vector normal to the surface of the REV	

$n_i$	Unit vector normal to the boundary	
$n_{rg}$	Exponent parameter for the saturation degree formulation	
$n_{rw}$	Exponent parameter for the saturation degree formulation	
$p$	Pressure	$ML^{-1}T^{-2}$
$p_0$	Initial pressure ( $t = 0$ )	$ML^{-1}T^{-2}$
$p_a$	Atmospheric pressure	$ML^{-1}T^{-2}$
$p_c$	Capillary pressure	$ML^{-1}T^{-2}$
$p_e$	Entry capillary pressure	$ML^{-1}T^{-2}$
$p_f$	Fracture pressure	$ML^{-1}T^{-2}$
$p_g$	Gas pressure	$ML^{-1}T^{-2}$
$p_g$	Virtual gas pressure	$ML^{-1}T^{-2}$
$p_g^f$	Fluctuation of gas pressure	$ML^{-1}T^{-2}$
$P_L$	Langmuir pressure	$ML^{-1}T^{-2}$
$p_m$	Matrix pressure	$ML^{-1}T^{-2}$
$p_w$	Water pressure	$ML^{-1}T^{-2}$
$p_w$	Virtual water pressure	$ML^{-1}T^{-2}$
$p_w^f$	Fluctuation of water pressure	$ML^{-1}T^{-2}$
$p_{g,f}$	Gas pressure in the fractures	$ML^{-1}T^{-2}$
$p_{g,m}$	Gas pressure in the matrix	$ML^{-1}T^{-2}$
$p_{g,m}^0$	Initial gas pressure in the matrix	$ML^{-1}T^{-2}$
$p_{g,m}^{lim}$	Limit gas pressure	$ML^{-1}T^{-2}$
$p_{g,m}^{max}$	Maximum gas pressure in the matrix	$ML^{-1}T^{-2}$
$p_g^{Ad,lim}$	Limit adsorbed gas pressure	$ML^{-1}T^{-2}$
$p_g^{Ad}$	Adsorbed gas pressure in the matrix	$ML^{-1}T^{-2}$
$p_g^{Ad,b}$	Adsorbed gas pressure in equilibrium with the fracture pressure	$ML^{-1}T^{-2}$
$P_{ij}$	First Piola-Kirchhoff stress tensor	$ML^{-1}T^{-2}$
$p_{rb}$	Rebound pressure	$ML^{-1}T^{-2}$
$p_{res}$	Reservoir pressure	$ML^{-1}T^{-2}$
$p_{res}^{crit}$	Critical reservoir pressure	$ML^{-1}T^{-2}$
$p_{w_0}$	Reference water pressure	$ML^{-1}T^{-2}$
$Q$	Source term	$ML^{-3}T^{-1}$
$q$	Flow	$LT^{-1}$
$q_f$	Flow between two parallel plates	$LT^{-1}$
$Q_g$	Gas source term	$ML^{-3}T^{-1}$

## LIST OF SYMBOLS

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$q_g$	Input gas flux	$[ML^{-2}T^{-1}]$
$q_i$	Flow vector	$[LT^{-1}]$
$q_L$	Longitudinal flow	$[LT^{-1}]$
$q_N$	Total flow through $N$ fractures	$[LT^{-1}]$
$q_T$	Transverse flow	$[LT^{-1}]$
$Q_w$	Water source term	$[ML^{-3}T^{-1}]$
$q_w$	Input water flux	$[ML^{-2}T^{-1}]$
$q_{g_i}$	Advective flow vector of the gas phase	$[LT^{-1}]$
$q_{g_L}$	Longitudinal flow of the gas phase	$[LT^1]$
$q_{g_T}$	Gas transverse flow	$[LT^{-1}]$
$q_{g_{well}}$	Mass gas production rate	$[ML^{-2}T^{-1}]$
$q_{l_i}$	Advective flow vector of the liquid phase	$[LT^{-1}]$
$q_{l_L}$	Longitudinal flow of the liquid phase	$[LT^1]$
$q_{w_{ob}}$	Income water mass flow on the outer boundary	$[ML^{-2}T^{-1}]$
$q_{w_{well}}$	Mass water production rate	$[ML^{-2}T^{-1}]$
$R$	Universal gas constant	$[ML^2N^{-1}\Theta^{-1}T^{-2}]$
$r$	Radius	$[L]$
$R_{ij}$	Rotation matrix	$[ - ]$
$s$	Coordinate along the channel	$[L]$
$S_g$	Gas mass storage term	$[ML^{-2}]$
$S_r$	Saturation degree	$[ - ]$
$S_r^*$	Normalized saturation	$[ - ]$
$S_w$	Water mass storage term	$[ML^{-2}]$
$S_{r,res}$	Residual saturation	$[ - ]$
$S_{r_g,res}$	Gas residual saturation degree	$[ - ]$
$S_{r_g}$	Gas saturation degree	$[ - ]$
$T$	Temperature	$[\Theta]$
$t$	Time	$[T]$
$T_i$	Projection of the local stress tensor in global coordinates	$[ML^{-1}T^{-2}]$
$t_i$	Traction vector	$[ML^{-1}T^{-2}]$
$T_t$	Transverse transmissivity of the fracture	$[M^{-1}L^2T^1]$
$T_{well}$	Transmissibility factor of the well	$[L^3]$
$u_i$	Displacement vector	$[L]$
$u_i^f$	Fluctuation displacement field	$[L]$
$u_i^e$	Equivalent displacement in the contact zone	$[L]$

$u_n$	Normal displacement	[L]
$u_n^{max}$	Maximal normal displacement allowed	[L]
$u_{l_k}$	Coordinate of the degree of freedom $l$ at node $k$	
$V$	Volume	[ $L^3$ ]
$v_i$	Velocity vector	[ $LT^{-1}$ ]
$v_i^*$	Admissible virtual velocity field	[ $LT^{-1}$ ]
$V_L$	Langmuir volume	[ $L^3 M^{-1}$ ]
$v_g$	Gas molecular velocity	[ $LT^{-1}$ ]
$V_g^{Ad}$	Adsorbed volume per unit of mass	[ $L^3 M^{-1}$ ]
$w$	Matrix width	[L]
$W_E^*$	External virtual work	[ $ML^2 T^{-2}$ ]
$W_G$	Gauss weight at the integration point $IP$	
$W_I^*$	Internal virtual work	[ $ML^2 T^{-2}$ ]
$w_{ij}$	Spin rate tensor	[ $T^{-1}$ ]
$X_i$	Coordinates in the reference configuration	[L]
$x_i$	Coordinates in the current configuration	[L]

### *Superscripts*

$[.]^e$	Quantity related to a finite element $e$
$[.]^F$	Quantity on the follow boundary
$[.]^L$	Quantity on the lead boundary
$[.]^M$	Macroscale quantity
$[.]^m$	Microscale quantity
$[.]^T$	Transposed object
$\dot{[.]}$	Time derivative
$^\circ[.]$	Quantity given in the orthotropic axes



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Dubito, ergo sum. Cogito, ergo sum.

René Descartes

